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SIMULTANEOUS MULTIPLE PARAMETER ADJUSTMENT IN ADAPTIVE SYSTEMS USING A SINGLE PERTURBATION SIGNAL

By

Kumpati S. Narendra and Theodore S. Baker

April 15, 1967

Control Theory

Dunham Laboratory Technical Report CT-11

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Summary+

A gradient method is presented for adjusting the parameters of a system to optimize a performance criterion. The partial derivatives of the performance index with respect to the adjustable parameters are obtained as real time signals, using a single perturbation signal for a system with unknown structure, but favorable topology.

Computer simulations are included to indicate the feasibility of the approach.

The adaptive control method using a single perturbation signal, as proposed here, has many features in commom with the well-known parameter perturbation scheme.

+ This paper will be presented at the Sixth Symposium on Adaptive Processes at the National Electronics Conference, Chicago, Illinois, October, 1967.

INTRODUCTION

A new perturbation method is presented in this paper for the simultaneous adjustment by the gradient method of multiple parameters in adaptive control systems. The significant advantage of this method is that only one perturbation signal is needed to generate the gradient of a performance criterion which is a function of all of the adjustable parameters. This results in a considerable saving in time and equipment over the conventional perturbation methods which use a separate perturbation for each parameter adjustment. At present the method has only been shown to work for systems with a favorable topology, where the adjustable gain parameters feed into a common summing junction that is linked with the system input by a single subsystem, as shown in Figure 1. This is not, however, necessarily overly restrictive since a controller with adjustable parameters often can be constructed with this structure even though the plant itself may not have a structure suitable for this perturbation method.

The single perturbation signal method is an outgrowth of the model methods of gradient generation [1, 2]. It was shown previously [2] that in a truly adaptive situation, where much of the structure of the system is unknown and a model cannot be constructed, that the system could be used as its own model by feeding back the appropriate error signal, attenuated and delayed beyond the autocorrelation time of the system signals. In this paper the idea of using the system as its own model is extended further by the demonstration that a multiplicative perturbation signal, which is uncorrelated with the major system signals, can be used in place of the time delay. Furthermore, it is shown that such conventional techniques as the well-known sinusoidal perturbation method

[3, 4] exhibit significant features in common with the method presented here.

First it is shown that a single perturbation signal can generate the signals necessary for the correlation technique construction of the gradient of the performance index in parameter space. Then this method is compared with the model method and the conventional perturbation methods. Next the design and stability of the adaptive loop is discussed and finally computer simulations which demonstrate the feasibility of the method are presented.

PERTURBATION METHOD

A performance criterion often used in adaptive procedures is the minimization of $\overline{e^2}$, where the bar denotes a statistical ensemble average, or a time average over a finite or infinite time interval. If k is an adjustable parameter, then the quantity required for the steepest descent adjustment of k is the gradient component

$$\frac{\partial}{\partial k}\left[\overline{e^2(t)}\right] = \overline{\frac{\partial}{\partial k}\left[e^2(t)\right]} = 2\,\overline{e(t)\frac{\partial e(t)}{\partial k}} \; .$$

The adaptive loop is an implementation of this correlation process. The error signal is multiplied by the partial derivative and the product is integrated and multiplied by a loop gain constant β . The resulting signal is subtracted from the gain k . The error $e(t)$ usually is available as an actual signal so that the gradient component for any particular k can be obtained by correlation of $e(t)$ and $\frac{\partial e(t)}{\partial k}$, if this partial derivative also can be generated as a real time signal. Even if a more general performance index, $J = \overline{F(e(t), t)}$, is used, the crucial part in determining the gradient is obtaining $\frac{\partial e(t)}{\partial k}$ as a signal.

It is shown now that if the system has several adjustable gain parameters k_i , ($i = 1, \ldots, m$) , then the partial derivatives $\dfrac{\partial e(t)}{\partial k_i}$ each appear at the inputs to the respective gains k_i . These desired signals are imbedded within the normal system signals at these points and must be extracted for use in the adaptive loops.

The convolution integral representations of the signals $e(t)$, $x(t)$, and $y_i(t)$ in the linear time invariant system shown in Figure 1 are

$$e(t) = \int_0^\infty H(s)\, e(t-s)\, ds = H{*}x$$

$$\begin{aligned} y_i(t) &= h_i{*}x \\ x(t) &= u + g{*}\Big(\sum_{i=1}^{m} k_i\, y_i + \varepsilon\rho e \Big) , \end{aligned} \qquad (1)$$

where H is the impulse response between the input summer output and the error output and h_i is the impulse response between the input summer output and the input to the gain k_i , with the output of every k_i open-circuited and with $\varepsilon\rho(t) = 0$. Also, $g(t)$ is the impulse response of the subsystem between the gain summer and the input summer.

The partial derivatives of each of these signals with respect to any one of the adjustable gains k_j is calculated from (1) as

$$\begin{aligned} \frac{\partial e}{\partial k_j} &= H{*}\frac{\partial x}{\partial k_j} \\ \frac{\partial y_i}{\partial k_j} &= h_i{*}\frac{\partial x}{\partial k_j} \\ \frac{\partial x}{\partial k_j} &= g{*}\Big(\sum_{i=1}^{m} k_i \frac{\partial y_i}{\partial k_j} + y_j + \varepsilon\rho \frac{\partial e}{\partial k_j} \Big) . \end{aligned} \qquad (2)$$

Since each impulse response corresponds to a linear time invariant system any sequence of convolutions can be permuted, for zero initial conditions. Therefore, equations (2) can be combined to yield

$$\frac{\partial e}{\partial k_j} = \left(g * \sum_{i=1}^{m} k_i h_i\right) * \frac{\partial e}{\partial k_j} + (g*H)*y_j + \varepsilon g*H*\left(\rho \frac{\partial e}{\partial k_j}\right). \quad (3)$$

If the magnitude of the perturbation signal, $\varepsilon\rho(t)$, is small compared with the magnitudes of the system signals, then the last term in (3) is small compared with the other terms.

Therefore, if a system were modeled by the integral equation representation

$$z_j = \left(g * \sum_{i=1}^{m} k_i h_i\right) * z_j + (g*H)*y_j \,, \quad (4)$$

then z_j would be the desired partial derivative $\frac{\partial e}{\partial k_j}$. It is shown now that (4) is an approximate representation for a portion of the signal $y_j(t)$, and that this portion can be extracted for use in generating the gradient of $\overline{e^2(t)}$.

Equations (1) can be combined to yield

$$y_j = \left(g * \sum_{i=1}^{m} k_i h_i\right) * y_j + \varepsilon\, g*h_j * \rho\,(H*x) + h_j*u \,. \quad (5)$$

Let $y_j = y_{j,0} + y_j'$ and $x = x_0 + x'$, where $y_{j,0}$ and x_0 are the signals developed when $\varepsilon\rho(t) = 0$. Then (5) becomes

$$y_j' = \left(g * \sum_{i=1}^{m} k_i h_i\right) * y_j' + \varepsilon\, g*h_j * \rho\,(H*x_0) + \varepsilon\, g*H_j * \rho\,(H*x'). \quad (6)$$

If $\varepsilon\rho(t)$ is small, then $x'(t)$ is much smaller than $x_0(t)$ and the last term in (6) can be ignored, which yields

$$y'_j = \left(g * \sum_{i=1}^{m} k_i h_i\right) * y'_j + \varepsilon\, g * h_j * \rho\,(H * x_0)\,. \qquad (7)$$

If $\rho(t)$ is slowly varying compared with the impulse response times of h_i and H then $\rho(t)$ can be shifted within the sequence of convolutions in the last term of (7) to yield $(g*H)*\varepsilon\rho y_{j,0}$. Therefore, (7) becomes identical to (4) except that the driving function in (7) is $\varepsilon\rho(t) y_{j,0}(t)$, whereas the driving function in (4) is $y_j(t)|_{\rho=0} = y_{j,0}(t)$. Because $\rho(t)$ is slowly varying the output of the system represented by (7) is approximately equal to the output of the system represented by (4) multiplied by $\varepsilon\rho(t)$, i.e., $y'_j \simeq \varepsilon\rho(t) z_j(t)$, or

$$y_j(t) = y_{j,0}(t) + \varepsilon\rho(t)\frac{\partial e(t)}{\partial k_j}\,. \qquad (8)$$

A heuristic demonstration that $\frac{\partial e}{\partial k_j}$ exists within the signal y_j is afforded by the model method. The system is shown with a model and without the perturbation signal, in Figure 2. The system equations are the same as (1), with $\varepsilon = 0$, and these can be combined to yield

$$\frac{\partial e}{\partial k_j} = \left(g * \sum_{i=1}^{m} k_i h_i\right) * \frac{\partial e}{\partial k_j} + (g*H) * y_j\,, \qquad (9)$$

which is the same as (3) with $\varepsilon\rho(t) = 0$.

The model equations are

$$\begin{aligned} f &= H*w \\ z_i &= h_i * w \\ w &= g*\left(\sum_{i=1}^{m} k_i z_i + e\right), \end{aligned} \qquad (10)$$

which can be combined with (1) to yield

$$z_j = \left(g * \sum_{i=1}^{m} k_i h_i \right) * z_j + (g * H) * y_j \quad . \tag{11}$$

For zero initial conditions (9) and (11) gener the same signal, so that

$$z_j = \frac{\partial e}{\partial k_j} \quad .$$

Now if $e(t)$ is multiplied by a low frequency perturbation signal $\varepsilon\,\rho(t)$ before insertion into the model, it is claimed that the new signals in the model will be approximately equal to the respective old signals, each multiplied by $\varepsilon\,\rho(t)$. Secondly it is claimed that the perturbation signal is not large enough to disturb the system significantly, and the system can be used as its own model.

ADAPTIVE SCHEME

As in most adaptive methods using the gradient technique, the basic idea in the present scheme is to construct the gradient of the performance index $\overline{e^2(t)}$ with respect to the control parameters, and then adjust these parameters in the direction of the negative gradient. In the model method the partial derivatives are generated directly as real time signals and $\frac{\partial \overline{e^2}}{\partial k_i}$ is obtained by multiplying $e(t)$ and $\frac{\partial e(t)}{\partial k_i}$ and integrating the product. The adaptive loop for the i^{th} gain is implemented as

$$k_i(t) = k_i(0) - \beta \int_0^t e(s) \frac{\partial e(s)}{\partial k_i} \, ds \quad , \text{ so that}$$

the integration for the continuous adjustment of k_i is combined with the integration for the averaging of $e^2(t)$.

In the present scheme $\frac{\partial e(t)}{\partial k_i}$ is imbedded in the signal $y_i(t)$, (8).

The extraction of the partial derivative and the correlation with $e(t)$ is incorporated within one integration as

$$k_i(t) = k_i(0) - \overline{\beta e \rho \; y_i} = k_i(0) - \beta' \; \overline{e \frac{\partial e}{\partial k_i}} , \qquad (12)$$

where $\beta' = \beta \varepsilon \overline{\rho^2}$. The simplification in (12) follows from (8) and the assumption that $\rho(t)$ is uncorrelated with the system signals and has zero mean.

In the usual parameter perturbation schemes for adaptive systems [3, 4] each gain k_i is perturbed by a signal $\rho_i(t)$ and the partial derivatives $\frac{\partial e}{\partial k_i}$ all are imbedded in the perturbed error signal. It is commonly assumed that $e^{*2} = e^2 + \sum_{i=1}^{m} c_i \rho_i \frac{\partial e^2}{\partial k_i} + n(t)$, where e^* is the perturbed error, e is the unperturbed error, and $n(t)$ is noise. The gradient is then extracted by correlation as

$$\overline{e^{*2} \rho_j} = \overline{\rho_j e^2 + \sum_{i=1}^{m} c_i \rho_i \rho_j \frac{\partial e^2}{\partial k_i}} = c_j \overline{\rho_j^2} \, \overline{\frac{\partial e^2}{\partial k_j}} ,$$

where the ρ_i have zero mean and are uncorrelated with each other. This extraction and correlation, however, can be formulated in the manner of the perturbation scheme of this paper, too. Thus

$$e^* = e + \sum_{i=1}^{m} c_i \rho_i \frac{\partial e}{\partial k_i} + n(t) ,$$

$$\overline{e^* (\rho_j e^*)} = \overline{(e + \sum_{i=1}^{m} c_i \rho_i \frac{\partial e}{\partial k_i})^2 \rho_j} = 2 c_j \overline{\rho_j^2} \, \overline{\frac{\partial e^2}{\partial k_j}} .$$

ADAPTIVE PARAMETERS AND STABILITY

The four adaptive parameters of a system using this new perturbation method are (i) the perturbation frequency, ω , (or period, T), (ii) the gain of the perturbation loop, ϵ , (iii) the gain of the adaptive loop, β , and (iv) the bandwidth of the adaptive loop. The parameter ϵ is adjusted so that the perturbed system does not differ markedly from the unperturbed system and the first term in the Taylor expansions are adequate for approximation purposes. Too small a value of ϵ , however, requires a long averaging time. If only a single integrator is used in the adaptive loop, then β determines the gain and bandwidth of the adaptive loop. If β is increased, then the speed of adaptation is increased but the effect of noise (due to averaging $\rho(t)\, e(t)\, y_1(t)$ over a short time) results in larger excursions of the parameters from their optimum values, as discussed below.

The choice of the perturbation frequency ω is decided by considerations of stability and accuracy. Generally, upper and lower bounds, ω_1 and ω_0 , respectively, can be found. If ω is greater than ω_1 then the approximations made while deriving the gradient signal are no longer valid. If ω is less than ω_0 then a large phase lag may be introduced in the loop, which causes instability . Slow perturbations involve longer averaging times which are equivalent to longer time delays in the feedback loop.

At the present time no general method exists for analyzing adaptive systems involving significant adaptive loop and system intercoupling. Therefore, for purposes of analysis the adaptive loop gain β which controls the speed of adaptation is assumed small. With this simplifying assumption a stability

analysis of the system using the error perturbation described in this paper, is carried out exactly as in the case of a parameter perturbation system.

Using the method suggested by Eveleigh [3] , if the index of performance can be expressed as

$$\overline{e^2(t)} = a_1(k - k_o)^2 + a_2 ,$$

for a single adjustable parameter k , then

$$\frac{\partial \overline{e^2(t)}}{\partial k} = 2 a_1(k - k_o) ,$$ where a_1 and a_2 are constants and k_o is the optimal value of k .

The signal used to adjust the parameter k is obtained by multiplying the two signals, $y(t) = y_o(t) + \varepsilon \rho(t) \frac{\partial e(t)}{\partial k}$, (8) , and $\rho(t) e(t)$.

If the product of these two signals is integrated over a sufficiently large time interval, then the first term $\theta(t) = \int_o^t y_o(t) \rho(t) e(t) dt$ becomes negligible because $\rho(t)$ is uncorrelated with the input or other system signals. The second term is

$$\int_o^t \frac{\varepsilon}{2} \rho^2(t) \frac{\partial e^2(t)}{\partial k} dt \simeq \cdot \varepsilon \gamma a_1 [k - k_o] ,$$

where $\int_o^\infty \rho^2(t) dt = \gamma$, the variance of the independent random perturbation signal.

If β is the gain of the feedback loop, the rate of change of the parameter k is given by

$$\dot{k} = - \beta \varepsilon \gamma a_1 (k - k_o) = - \alpha (k - k_o) ,$$ where $\alpha = \varepsilon \beta \gamma$. It is seen that in the ideal case the parameter k varies exponentially to reach the final value k_o . In practice, however, the averaging

time is not sufficiently large to reduce $\theta(t)$ to zero and this term acts as an added forcing function so that,

$$\dot{k} + \alpha k = \alpha k_o + \theta(t),$$

and the final value of $k(t)$ oscillates about the optimal value k_o.

In the multiple parameter case the same argument is used for each of the adjustable parameters $k_1, k_2 ---, k_n$. If $\overline{e^2(t)}$ in the parameter space is represented by

$$\overline{e^2(t)} = [\underline{k} - \underline{k}_o]^T \; P \; [\underline{k} - \underline{k}_o] + \underline{a} \quad ,$$

where k is the parameter vector, $\underline{a}$ is a constant vector and P is a positive definite matrix, then the simplified adaptive loop is described by the vector equation

$$\dot{k} = -\beta \varepsilon \gamma P [k - k_o] + \underline{\theta}(t) \quad ,$$

where $\underline{\theta}(t)$ is obtained by the finite time averaging of $\int_o^t \rho(t) e(t) \underline{y}_o(t) dt$.

In the ideal case $\underline{k}(t)$ approaches $\underline{k}_o$ exponentially. If $\underline{\theta}(t)$ is nonzero, the final value of $\underline{k}(t)$ oscillates in the vicinity of $\underline{k}_o$.

COMPUTER SIMULATIONS

Systems modeled by difference equations have been simulated on the 7094/7090 digital computer at Yale University to illustrate the single perturbation signal method of adaptation. The error function in each case is the difference between the output of the system and the output of a reference system which is fed by the same input. In order to demonstrate the feasibility of the method the reference system and the actual system are given the same structure so that the actual system can "identify" the reference exactly, in the absence of noise. Examples are presented for two and three adjustable parameters with sinusoidal and white noise perturbation signals, and with additive white noise.

The adaptive scheme used for adjusting parameters A, B, and C is shown in Figure 3a. The adjustable parameter system has a pulse transfer function $\dfrac{z^3}{z^3 + Az^2 + Bz + C}$, and the reference system has a pulse transfer function $\dfrac{z^3}{z^3 - 0.1z^2 + 0.2z - 0.25}$. In examples 1 and 3 the parameter C is fixed at -0.25, and the region in the (A, B) parameter space where the system is asymptotically stable is shown in Figure 3b .

The input to all systems in the following examples is a white noise signal with uniform distribution between -0.5 and +0.5. This input is a sequence of random numbers (generated by a computer program) whose statistical properties have been verified by direct computer calculation. Furthermore, both the same random number sequence at the input and the same initial settings of the adjustable parameters (zero) have been used throughout to provide a basis

for comparison among the different situations.

In all of these examples the performance index for implementing the adaptation is the continuous adjustment of the parameters with the instantaneous integral of the square error. The performance index for evaluating the adaptive scheme, however, is more meaningful as the mean square error. A theoretical mean square error criterion is most easily used in calculations if the integration interval is infinite. Therefore, one index reported in all of these results is the mean square error at any time t,

$$mse_1 = \frac{1}{t} \int_0^t e^2(s)\,ds = \frac{1}{t} \sum_{i=1}^{t} e^2(i) .$$

As time increases, the values of this index approach those for an infinite time average, but even for large integration intervals this index depends significantly on the initial state of the system. Therefore, a second index reported in all of these results is the square error averaged over only the previous 150 units of time, at any time t,

$$mse_2 = \frac{1}{150} \int_{t-150}^{t} e^2(s)\,ds = \frac{1}{150} \sum_{i=0}^{150} e^2(t-i) .$$

Since 150 units of time is much larger than the impulse response time of the reference system, (which is approximately 5 units of time), or than the correlation times of any signals used here, the index mse_2 is a good measure of the closeness of any set of parameters to their optimum value, for large time.

<u>EXAMPLE 1</u>:

In this example the perturbation is a sinusoid of unit amplitude and period, T. For a specific setting of the adaptive parameters, the time responses of the

adjustable parameters A and B are shown by the solid curves in Figure 4b . After 2000 units of time A and B have reached the values -0.1 and $+0.2$, respectively and the system is indistinguishable from the reference. This is corroborated by the reduction of mse_2 from 5×10^{-3} to 5×10^{-7} , as shown by the solid curves in Figure 4c . After 5000 units of time mse_2 is zero within the accuracy of the computer.

It has been observed in other test runs that the adjustable parameters approach their optimum values for wide ranges of β and ϵ and for periods of the sinusoidal perturbations ranging between 10 and 60 units of time (which is large compared with the impulse response time of 5 units for the reference system) .

If additive white noise is inserted at the system output, then the adaptation is slower and subject to more fluctuations, as shown by the dotted curves in Figures 4b, c . Nonetheless, the system adapts nearly as well as is possible , as shown by the eventual approach of mse_2 to the experimentally determined value for a system with parameters fixed identically to those of the reference system.

EXAMPLE 2 :

For the same setting of β and ϵ , but with the sinusoidal perturbation at twice the frequency of that in Example 1, all three parameters, A , B , and C adjust near to their optimum values within 3000 units of time, as shown by the dotted curves in Figures 5b, c . During the next two thousand units of time, however, there are drastic fluctuations. For these settings of β and T , but with ϵ reduced, the system adapts at almost the same speed and no

instability occurs, as shown by the solid curves in Figures 5b, c .

This stable case with three parameters corroborates the claim that multiple parameters can be adjusted simultaneously with a single perturbation signal. The fluctuating case also supports this contention but emphasizes the dependence of the scheme on the settings of the adaptive parameters. The most likely explanation for the instability with larger ε is that this increase in the perturbation signal disturbs the system sufficiently so that an incorrect gradient is generated in the vicinity of the optimum settings. Thus the adaptive system hunts around the optimum settings.

EXAMPLE 3 :

Despite the emphasis on low frequency perturbation signals it is shown in Figures 6, 7 that the system parameters adjust along the gradient even if the perturbation signal is broad band noise. It is seen by comparing Figure 6 with Figure 4, for two adjustable parameters, and Figure 7 with Figure 5, for three adjustable parameters, that the adaptation is much slower and less accurate for the white noise perturbation than for the low frequency sinusoid. Yet the fact that the perturbation noise is uncorrelated with the input noise permits the adaptive loop to utilize the low frequency content of the perturbation signal in the extraction and correlation process (12) , for producing the gradient. As expected the values of β and ε must be increased to compensate for the loss of high frequency perturbation signal energy.

Work is in progress to determine experimentally and analytically the behavior of these systems for other sets of initial conditions and for wider ranges of the adaptive parameters.

CONCLUSION

To the best of the authors' knowledge this is the first demonstration of an adaptive scheme for adjusting many parameters simultaneously with one perturbation signal, without the necessity of using a model or an equivalent system identification scheme. The method is limited at present to situations where the adjustable gain parameters feed into a common summing junction, but this limitation is not overly restrictive in the design of a controller. It is expected that the method can be extended to certain time-varying and non-linear situations.

REFERENCES

1. K. S. Narendra and L. E. McBride, Jr., "Multiparameter Self-Optimizing Systems Using Correlation Techniques," IEEE Trans. on Automatic Control, AC-9, 31-38, January 1964.

2. K. S. Narendra, M. A. L. Thathachar, and T. S. Baker, "Adaptive Control Using Time Delay and Correlation," Proceeding of the Third Annual Allerton Conference, University of Illinois, Urbana, Illinois, 1965, pp. 130-143.

3. V. W. Eveleigh, "General Stability Analysis of Sinusoidal Perturbation Extrema Searching Adaptive System," Proceedings of the Second International Conference, IFAC, Automatic and Remote Control, 1963, pp. 472-481.

4. R. J. McGrath and V. C. Rideout, "A Simulator Study of a Two-Parameter Adaptive Control System," IRE Trans. on Automatic Control, AC-6, 35-42, February 1961.

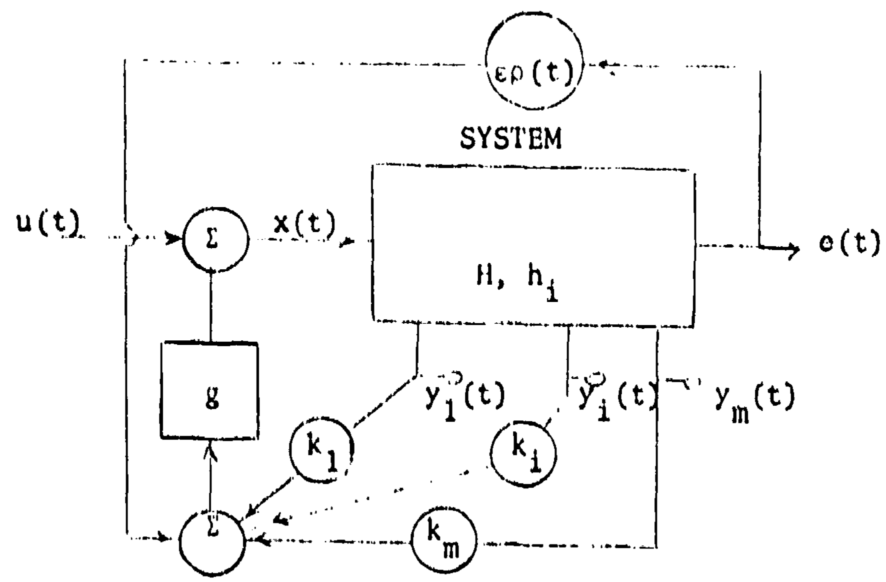

FIGURE 1. Single Perturbation Signal Scheme for Gradient Generation

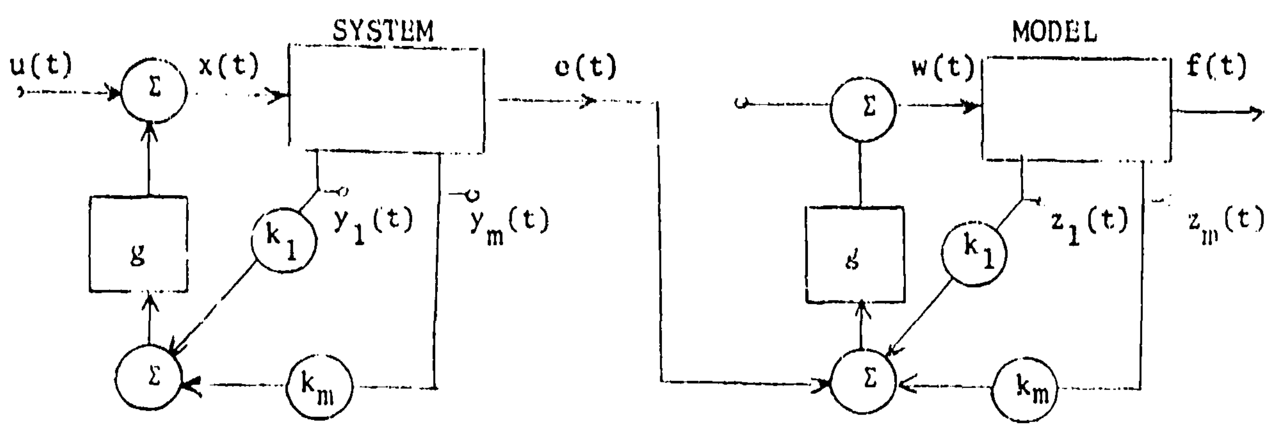

FIGURE 2. Model Method Scheme for Gradient Generation

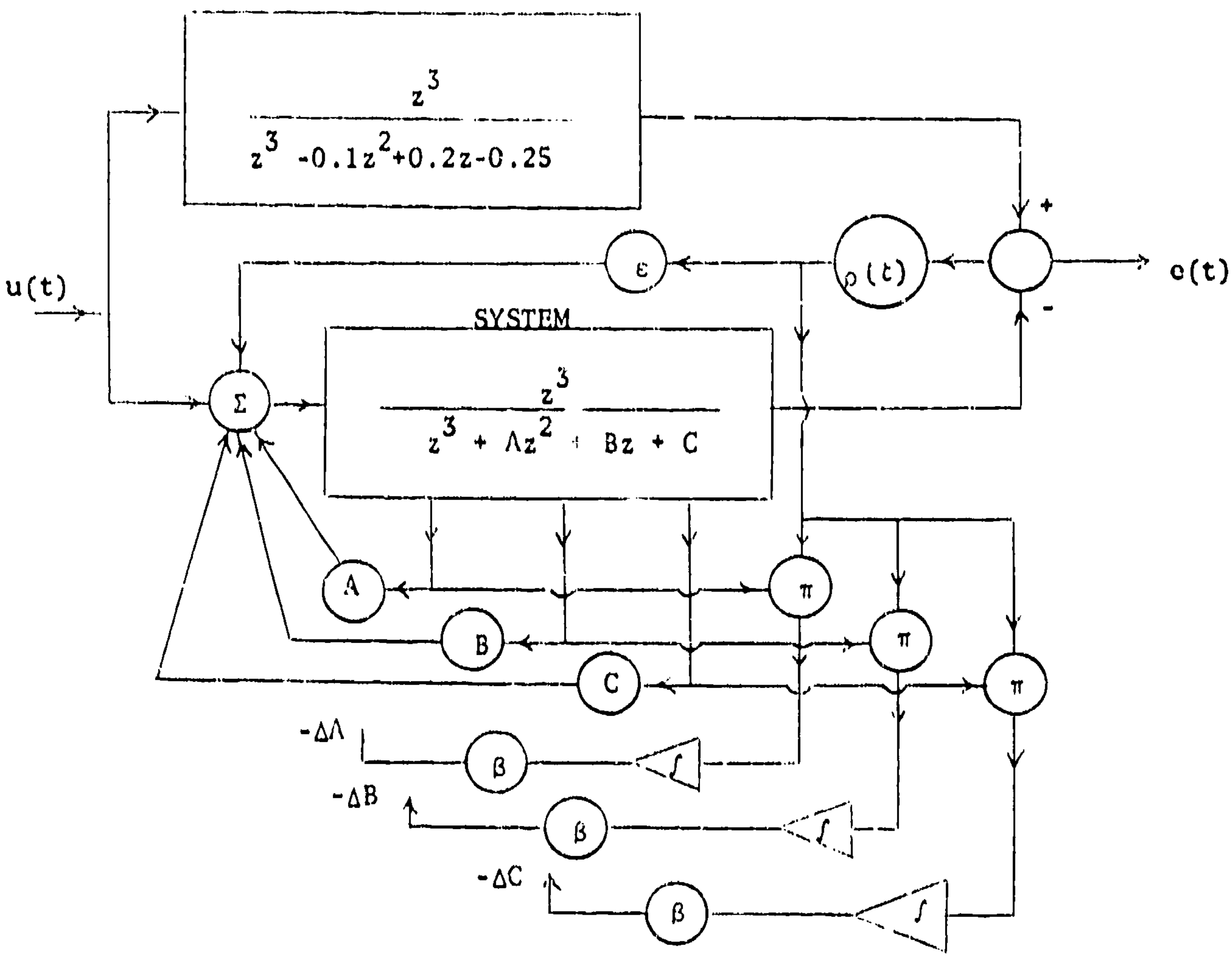

FIGURE 3a. Adaptive Scheme

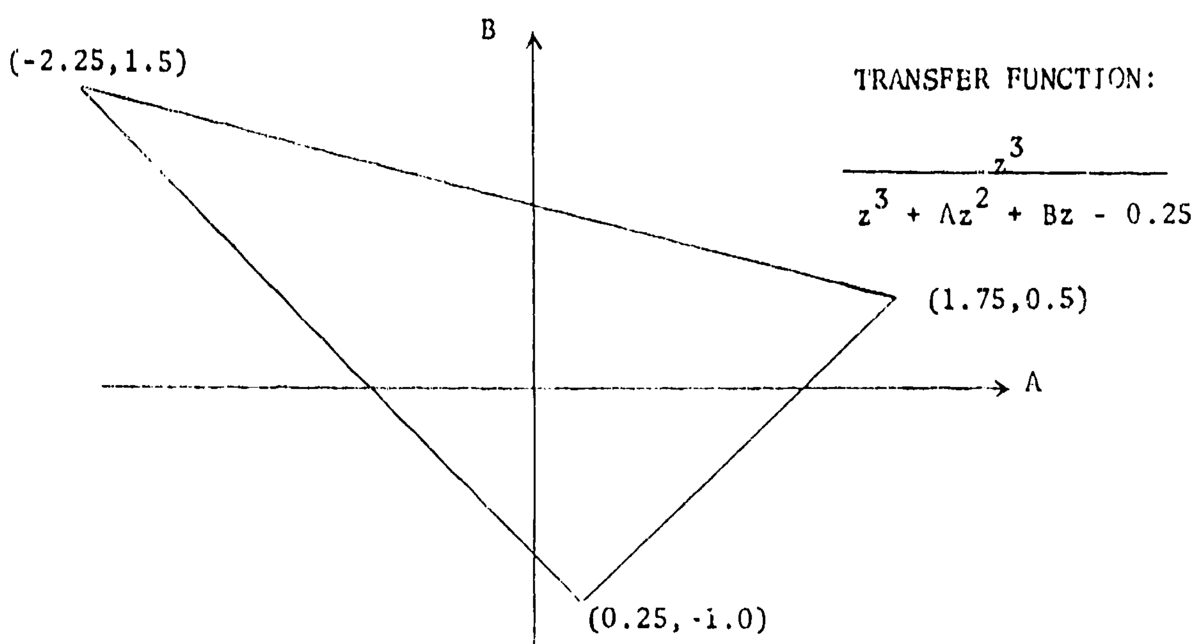

FIGURE 3b. Stability Region in (A,B) Parameter Space

$\beta = 0.3$, $\varepsilon = 0.3$

$\rho(t) = \sin \frac{2\pi}{T} t$, $T = 20$

Additive Noise

———————: zero

- - - - - - - - - -: uniform distribution, ± 0.05

Figure 4a. Adaptive Parameters

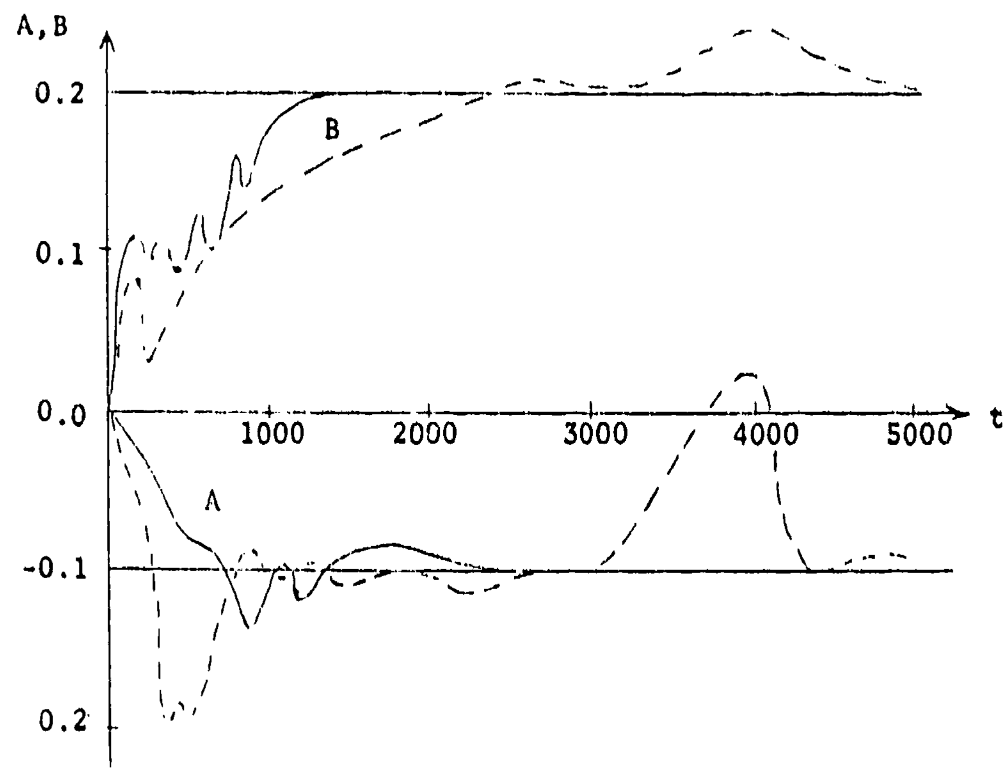

Figure 4b. System Parameters

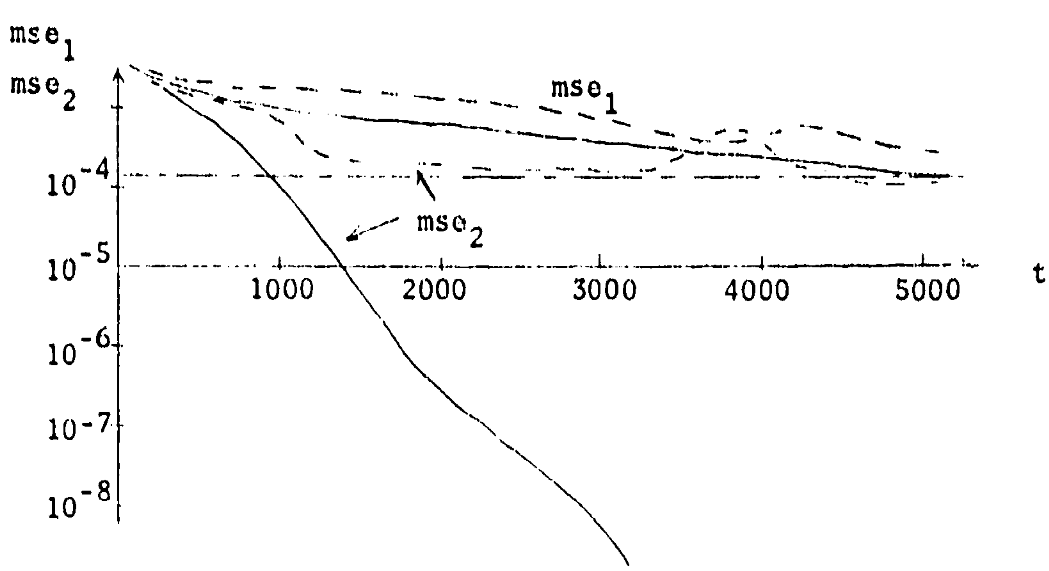

Figure 4c. Mean Square Errors

Figure 4. Adaptation with Single Sinusoidal Perturbation Signal

(Two Adjustable Parameters)

———————— : $\beta=0.3$, $\varepsilon=0.1$

- - - - - - - - - - : $\beta=0.3$, $\varepsilon=0.3$

$$\rho(t) = \sin \frac{2\pi}{T} t, \quad T=10$$

FIGURE 5a. Adaptive Parameters

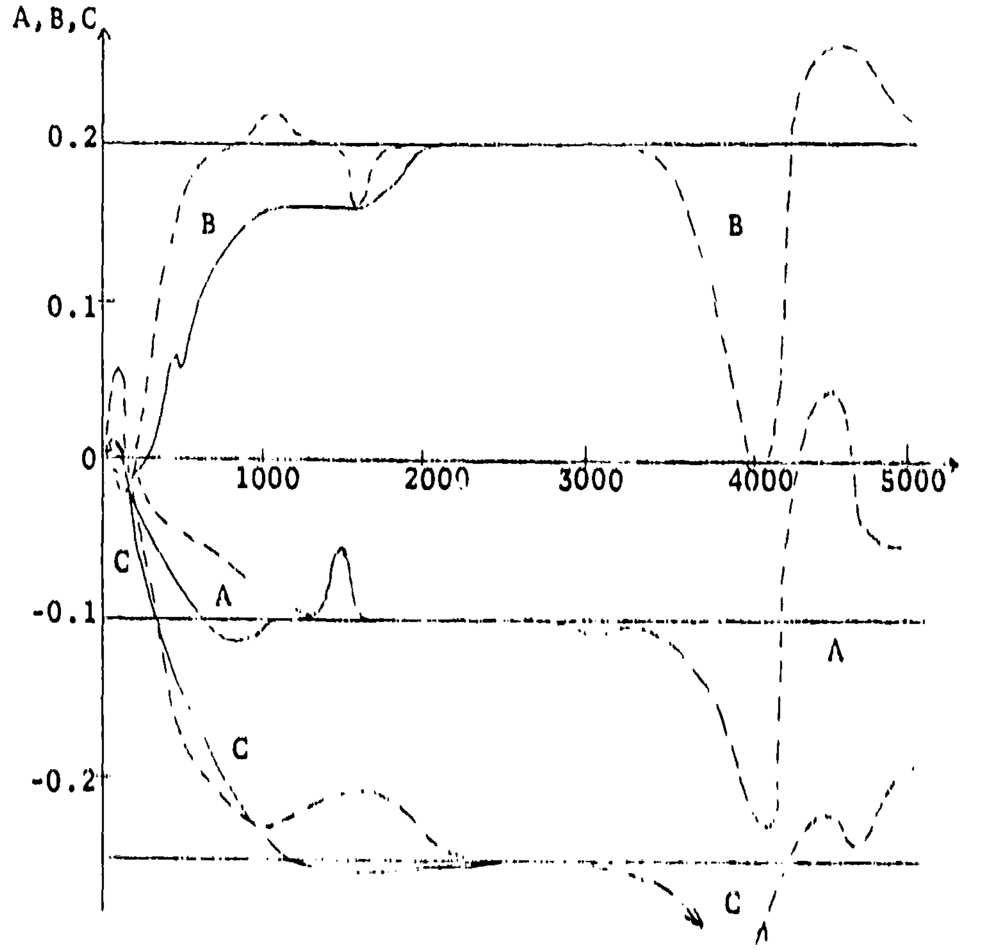

FIGURE 5b. System Parameters

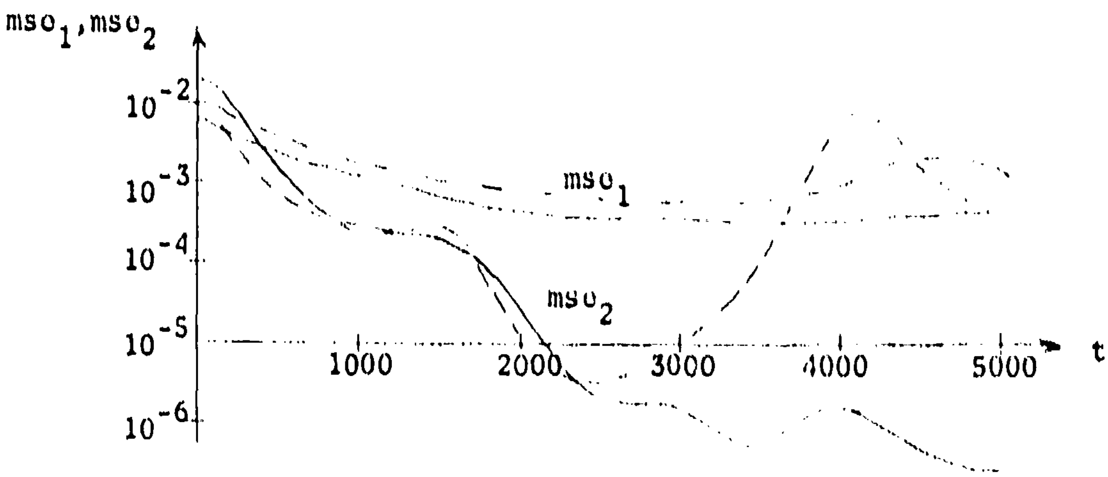

FIGURE 5c. Mean Square Errors

FIGURE 5. Adaptation with Single Sinusoidal Perturbation Signal

(Three Adjustable Parameters)

β=0.5, c=0.5

$\rho(t)$: White Noise, Uniform Distribution, ±0.5

FIGURE 6a. Adaptive Parameters

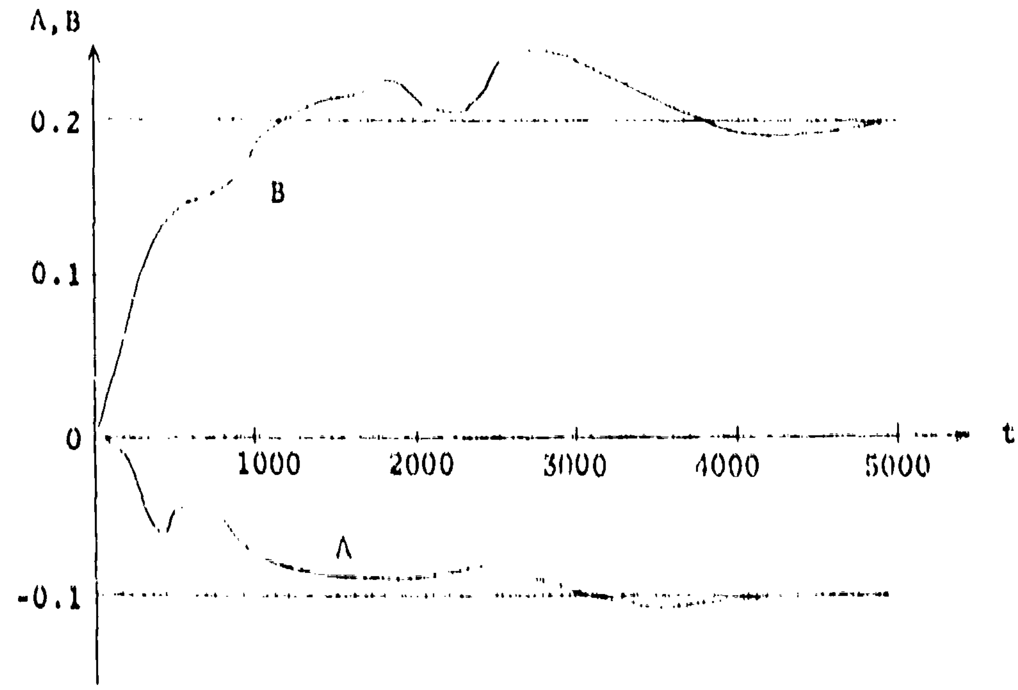

FIGURE 6b. System Parameters

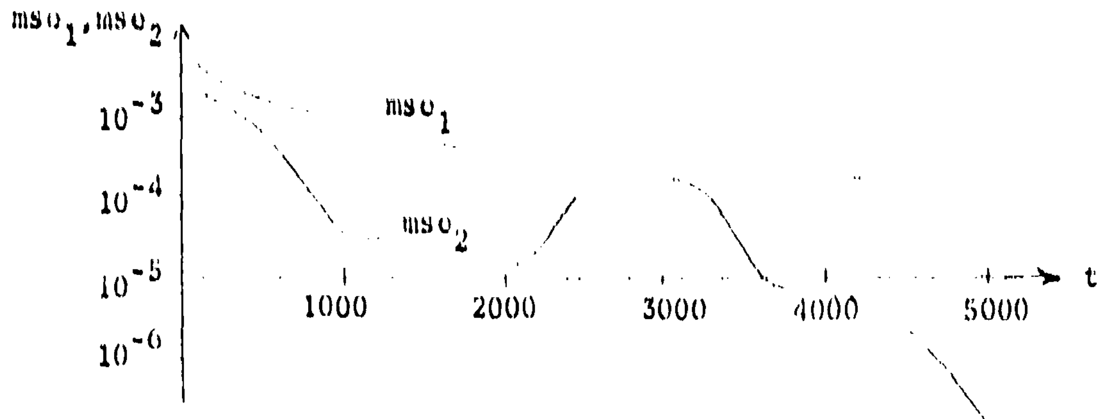

FIGURE 6c. Mean Square Errors

FIGURE 6. Adaptation with White Noise Perturbation Signal

(Two Adjustable Parameters)

$\beta = 0.4$, $\varepsilon = 0.4$

$\rho(t)$: White Noise, Uniform Distribution, ±0.5

FIGURE 7a. Adaptive Parameters

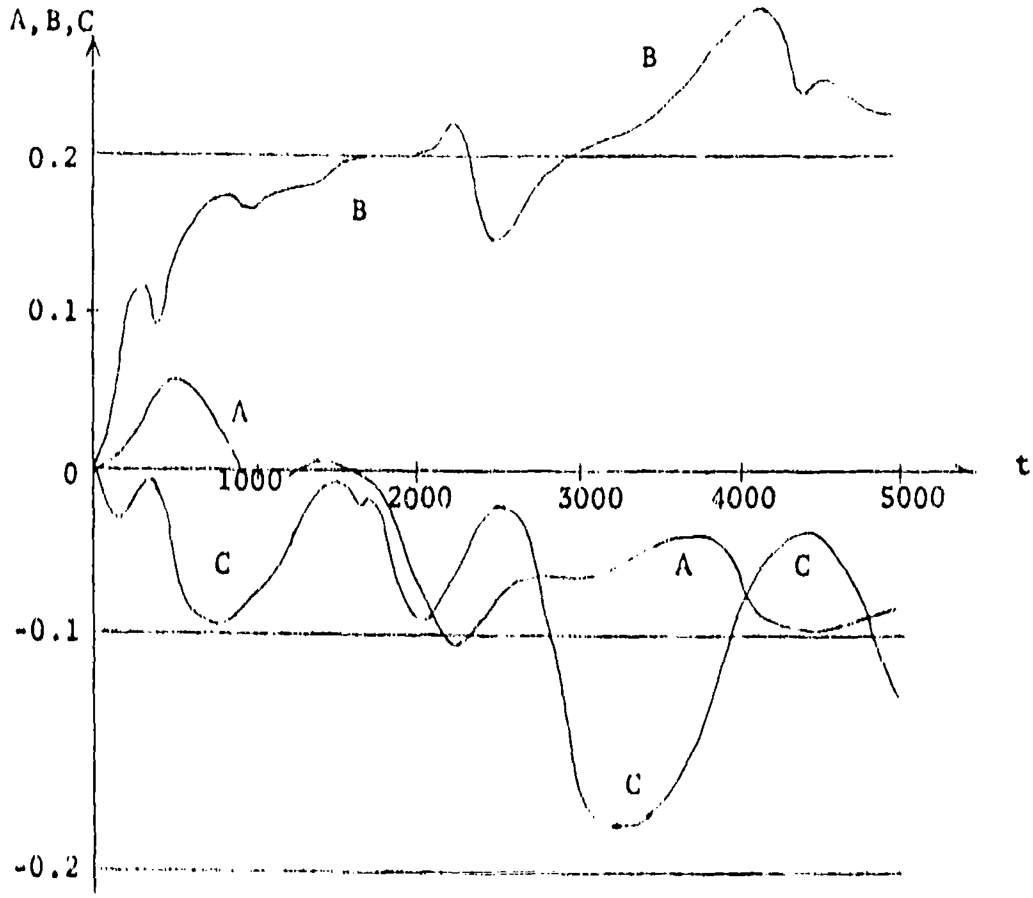

FIGURE 7b. System Parameters

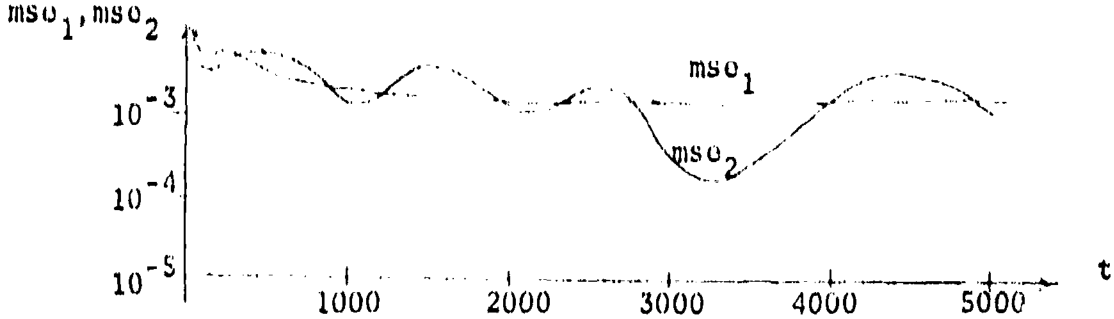

FIGURE 7c. Mean Square Errors

FIGURE 7. Adaptation with White Noise Perturbation Signal (Three Adjustable Parameters)

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